Reservation Rate, Risk and Equilibrium Credit Rationing

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Abstract

If prospective borrowers are capital constrained and can postpone their loan applications, the sign of reservation rate-risk relationship depends on the gearing ratio required to carry out investment. The paper shows that equilibrium credit rationing, in the sense of Stiglitz and Weiss (1981), can emerge only at sufficiently high gearing ratios.

JEL Classification: D82; D92; G21; G31

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1 Introduction

Stiglitz and Weiss (1981) demonstrate that, under imperfect information, an increase in lending interest rate may adversely affect the risk composition of the pool of loan applicants, which, in turn, may deteriorate banks' expected returns.[1] As a result, credit market may not clear as banks may resort to random rejection of seemingly identical loan applicants, even though the latter are willing to pay all the price and non-price elements of loan contracts. This phenomenon is termed Type II (or pure) credit rationing.[2]

A necessary condition for equilibrium credit rationing, in the sense of Stiglitz and Weiss (1981), to arise is that the relationship between the maximum interest rate prospective borrowers are willing to pay (reservation rate) increases in the riskiness of their projects, *i.e.*, reservation rate-risk relationship is positive. Positive relationship means that higher risk investors are willing to accept higher cost of debt (interest rate) relative to their less risky counterparts. If the reservation rate-risk relationship is negative, the adverse effect of interest rate increase on the risk composition of the pool of loan applicants does not arise. Consequently, a simple interest rate adjustment clears the market for loans.

Lensink and Sterken (2001, 2002) question the existence of the positive reservation rate-risk relationship. By allowing prospective borrowers to postpone loan application until exogenous uncertainty (associated with the future state of nature) is *completely* resolved, they show that riskier investors have a greater propensity to wait before embarking upon an irreversible capital outlay and hence have lower reservation rates than their less risky counterparts.[3] As a result, reservation rate-risk relationship is negative and equilibrium credit rationing, in the sense of Stiglitz and Weiss (1981), does not arise.

This paper shows that, if waiting does not completely resolve uncertainty, the result of Lensink and Sterken holds only for low gearing ratios. For high gearing

[1] Arnold and Riley (2009) challenge the possibility of globally hump-shaped lender's expected return function that underpins the analysis of Stiglitz and Weiss. However, Agur (2012) demonstrates that it is attainable, if banks are funding constrained, and hence equilibrium credit rationing is possible.

[2] Type I credit rationing arises when some borrowers obtain smaller loan amounts than they demand (Jaffee and Modigliani, 1969).

[3] de Meza and Webb (2006) derive a model in which prospective borrowers can postpone their loan application and thereby signal their type. In their model, the timing of loan application plays a similar separating role as collateral, as described by Bester (1985, 1987). In our model, however, endogenous uncertainty (that banks face in relation to the loan applicant type) is not resolved with time and hence signalling, in the sense of de Meza and Webb, is not possible.

ratios, reservation rate-risk relationship is positive and the analysis of Stiglitz and Weiss is applicable. The threshold gearing ratio, above which reservation rate-risk relationship is positive, is derived explicitly.

The rest of the paper is organised as follows. Section 2 sets up the model. Section 3 demonstrates the positive reservation rate-risk relationship for projects without an option to postpone loan application. In section 4, the option to postpone loan application is incorporated into the model, and the threshold gearing ratio, above which Stiglitz and Weiss (1981) results hold, is derived. Section 5 concludes.

2 The model

Consider a three-period economy $t = 0, 1, 2$ comprised of risk-neutral banks and a pool of heterogeneous risk-neutral investors, or prospective borrowers, indexed by i. Each prospective borrower holds an irreversible and indivisible investment project. Given the risk-neutrality of all agents in the economy, they discount their future cash flows at the risk-free rate of return that is normalised at 0. Investors have the same initial wealth $W_{i,0} = W$ for all i, and the same investment capital requirements of $I_i = I$ for all i. Thus, to undertake their respective investment projects they need to obtain external financing in amount $L = I - W = gI$ for all i, where $g \in (0, 1)$ denotes the resulting gearing ratio. Capital markets are assumed to be segmented so that the only source of external financing is the non-recourse bank loan (m, L, q), where $m > 0$ denotes the interest rate charged by banks, and $q \in (0, 1]$ is the probability of obtaining a loan, if application is made.[4] m is greater than the risk-free rate to compensate banks for the risk of default of loan applicants. q accounts for the potential rationing equilibrium in credit market so that even though loan applicants are willing to pay all the price and non-price elements of loan contracts offered, not all applicants obtain the loan. The two limiting cases of q are: $q = 1$ that pertains to a situation wherein credit rationing is absent, and $q = 0$ that results from credit market collapse.[5]

Payoffs of projects held by investors are uncertain. A project can fail at both $t = 1$ and $t = 2$ due to an unfavourable realisation of the cash flow process. If project fails in a given period, its unlevered cash flow from that period onwards

[4]Under loan contract, borrower pledges the entire project to lender as collateral. Additional collateral requirement is set at 0.

[5]An example of credit market collapse is provided by Mankiw (1986).

is 0. The probability that project held by investor i does not fail at $t = 1$ is $p_i \in (0, 1)$, in which case investor receives u_i of unlevered cash flow at $t = 1$. The probability that project does not fail at $t = 2$ conditional on having not failed at $t = 1$ is p_i^2, in which case investor i receives u_i^2 of unlevered cash flow at $t = 2$ in addition to the cash flow she has obtained at $t = 1$. Projects are assumed to differ by mean-preserving spreads (Rothschild and Stiglitz, 1970), and yield the same expected present value of future unlevered cash flows. Hence, if investor i undertakes her project at $t = 0$, the present value of unlevered cash flows is given by:

$$p_i u_i + p_i^2 u_i^2 = F, \quad \text{for all } i, \tag{1}$$

where:

$u_i = \frac{\sqrt{1+4F}-1}{2p_i} > 0$, and $\frac{\mathrm{d}u_i}{\mathrm{d}p_i} = -\frac{u_i}{p_i} < 0$.

All projects are assumed to yield a positive unlevered net present value (NPV) at $t = 0$ so that $F > I$, and hence they all are socially desirable. Although banks know that loan applicants' projects differ by their risk characteristics, they can only ascertain F and I, but cannot observe p_i and u_i. As a result, they cannot tailor loan contract terms and conditions to a given investor's risk profile and offer a single loan contract to all loan applicants.

3 Projects without an option to postpone loan application

Suppose prospective borrowers can apply for loans only at $t = 0$. Given the loan contract (m, L, q), the value of investor i's equity claim in a completed project (*i.e.*, after the loan has been obtained and investment cost incurred) is:

$$E_{i,0}(m, L) = F - p_i m L - p_i^2 (1 + m) L, \quad \text{for all } i. \tag{2}$$

The expected present value of a project held by investor i prior to incurring capital outlay at $t = 0$ is then:

$$q \left(E_{i,0}(m, L) - W\right)_+, \quad \text{for all } i, \tag{3}$$

where:

$(x)_+ = \max[x, 0]$.

In this setting, investor i uses a Marshallian decision rule, and applies for loan only if:

$$E_{i,0}(m, L) \geqslant I - L, \quad \text{for all } i. \tag{4}$$

Solving inequality (4) for m, we obtain the maximum interest rate a given loan applicant can afford without abandoning its investment programme, *i.e.*, investor i's reservation rate for a project without an option to postpone loan application:

$$m_i^N = \frac{F - I}{p_i (1 + p_i) gI} + \frac{1 - p_i}{p_i}, \quad \text{for all } i. \tag{5}$$

Investor i applies for loan only if $m \leqslant m_i^N$.

If banks could perfectly assess the risk profile of each loan applicant, they would charge a given investor i interest rate m_i^C that is determined by their zero expected profit conditions:[6] $m_i^C = \frac{1-p_i}{p_i}$, for all i. Since $m_i^C < m_i^N$ for all i, all investors apply for loans at $t = 0$, and all socially desirable projects are undertaken.

Under imperfect information, banks charge all borrowers the same interest rate m, and hence are interested in how their standardised (blanket) lending policy affects the risk composition of the pool of loan applicants. More specifically, they are interested in knowing whether loan applicants' pool is comprised of the lower or higher risk investors. The solution to this problem is achieved by verifying how reservation rate changes across the pool of investors. If reservation rate decreases in riskiness of the underlying project, then an increase in lending rate will result in a monotonic increase in the expected return per loan to the bank. As a result, market for loans will always clear. On the other hand, if reservation rate increases in the risk of the underlying project, then expected return per loan to the bank may be a humped function of interest rate. As a result, market for loans may not clear and equilibrium credit rationing may emerge.

Differentiating investor i's reservation rate with respect to p_i gives:

$$\frac{\mathrm{d}m_i^N}{\mathrm{d}p_i} = -\frac{m_i^N (1 + 2p_i) + 2p_i}{p_i (1 + p_i)}. \tag{6}$$

[6] If banks can access funding at the risk-free rate, then a given bank's zero expected profit condition ensures that the amount lent at $t = 0$ equals the expected return from extending a loan to investor in particular risk category p_i:

$$L = p_i m_i^C L + p_i^2 \left(1 + m_i^C\right) L, \quad \text{for all } i.$$

$\frac{\mathrm{d}m_i^N}{\mathrm{d}p_i} < 0$ for all i, *i.e.*, reservation rate of investors holding projects without an option to postpone loan application decreases in probability of success. Hence, the least risky investors have the lowest reservation rate. This result can be formulated in terms of the analysis of Stiglitz and Weiss (1981). For a given interest rate m, there is a critical level of probability of success such that investors apply for loans only if their probability of success is below this critical level. Furthermore, as interest rate increases, the critical probability level decreases and the pool of loan applicants becomes riskier.

4 Projects with an option to postpone loan application

Suppose prospective borrowers can apply for loans at $t = 0$, or postpone their loan applications until $t = 1$. In effect, each prospective borrower holds an option to postpone her loan application and hence investment. By postponing loan application, investor gains additional information about the project and invest only if the realisation of the cash flow process is favourable. Waiting, however, is costly, because investor forgoes expected period $t = 1$ cash flow: $p_i (u_i - mL)$ for all i.

The value of equity claim in a completed project embarked upon at time $t = 1$ is:

$$E_{i,1}(m, L) = p_i u_i^2 - p_i (1 + m) L, \quad \text{for all } i. \tag{7}$$

If investor i decides to postpone loan application until $t = 1$, she obtains the continuation value of the project she holds. Its expected present value at $t = 0$ is:

$$p_i q (E_{i,1}(m, L) - W)_+, \quad \text{for all } i, \tag{8}$$

Notice that, by postponing loan application until $t = 1$, investor cannot remove *all* uncertainty associated with her project's payoff. Therefore, projects that are embarked upon at $t = 1$ are subject to default risk.[7] As a result, lending interest rate is strictly above the risk-free rate, and, under imperfect information, a credit rationing equilibrium at $t = 1$ may emerge.

The flexibility afforded by the option to wait implies that there is a trade-off between the value of immediate loan application and the expected present value

[7] This is the key distinction of our model in comparison to the model of Lensink and Sterken (2002), where default risk is eliminated at $t = 1$.

of continuation value. The overall value of the project for investor i is therefore:

$$\max\left[q\left(E_{i,0}\left(m,L\right)-W\right)_{+},p_{i}q\left(E_{i,1}\left(m,L\right)-W\right)_{+}\right],\quad \text{for all } i. \tag{9}$$

The value of option to postpone loan application is the additional value investor i obtains from waiting until $t=1$ rather than applying for loan immediately at $t=0$. Let $V_i\left(m,L\right)$ denote the value of option to postpone loan application by investor i. It is given by the difference between the continuation value and intrinsic value of the underlying project:

$$\begin{aligned} V_i\left(m,L\right) &= q\left[p_i\left(E_{i,1}\left(m,L\right)-W\right)-\left(E_{i,0}\left(m,L\right)-W\right)\right] \\ &= q\left[p_i^2u_i^2-p_i^2mL-p_i^2L-p_iW-F+p_imL+p_i^2mL+p_i^2L+W\right] \\ &= q\left[\left(1-p\right)W-p_i\left(u_i-mL\right)\right],\quad \text{for all } i. \end{aligned} \tag{10}$$

Investor i applies for loan at $t=0$, if $V_i\left(m,L\right)\leqslant 0$. Solving this inequality for m, we obtain the maximum interest rate a given loan applicant can afford without postponing her loan application, *i.e.*, investor i's reservation rate for a project with an option to postpone loan application. The following expression results:

$$m_i^R=\frac{p_iu_i-\left(1-p_i\right)I}{p_igI}+\frac{1-p_i}{p_i},\quad \text{for all } i. \tag{11}$$

Notice that although m_i^R may be greater than m_i^C for some i, m_i^R does *not* necessarily exceed m_i^C for all i. Hence, even under perfect information, it may be optimal to postpone some projects. More specifically, only investors with projects that have probability of success $p_i\geqslant p_i^C=\frac{1+2I-\sqrt{1+4F}}{2I}$ apply for loans at $t=0$ and obtain them with certainty.

Under imperfect information, banks charge all borrowers the same interest rate m. As previously, they can verify the composition of the loan applicants' pool by analysing how reservation rates change across the pool of prospective borrowers. Differentiating the reservation rate m_i^R with respect to the probability of success gives:

$$\frac{\mathrm{d}m_i^R}{\mathrm{d}p_i}=\frac{\left(1-g\right)I-p_iu_i}{p_i^2gI}. \tag{12}$$

Notice that $\frac{\mathrm{d}m_i^R}{\mathrm{d}p_i}<0$ if

$$g>g^*\equiv 1-\frac{\sqrt{1+4F}-1}{2I}. \tag{13}$$

Therefore, threshold gearing ratio g^* separates the range of gearing ratios into two regions. For gearing ratios above g^* *higher* risk borrowers have higher reservation rates, whereas for gearing ratios below g^* *lower* risk borrowers have higher reservation rates.

The analysis can be formulated in terms of critical risk levels (probabilities of success) as in Stiglitz and Weiss (1981). It is depicted in Figure 1. Let $p^*(m,g) = \frac{1-g-\frac{\sqrt{1+4F}-1}{2I}}{1-g(1+m)}$ denote the critical level of risk, at which the option to postpone loan application is worthless (*i.e.*, $V_i(m,g) = 0$) and thus there is no additional gain from waiting. Consider gearing ratio $g = g_1 > g^*$, for which the critical level of risk is $p^*(m,g_1) \equiv p_L$. Prospective borrowers with probability of success lower than p_L have reservation rates higher than m and hence apply for loans at $t = 0$, whereas investors with probability of success above p_L have reservation rates lower than m and postpone their loan applications. Hence, in the region $g > g^*$, the analysis of Stiglitz and Weiss holds. More specifically, for a given interest rate m, investors apply for loans if their probability of success is below the critical probability p_L, and, as the interest rate increases, the critical probability level decreases and the pool of loan applicants becomes riskier.

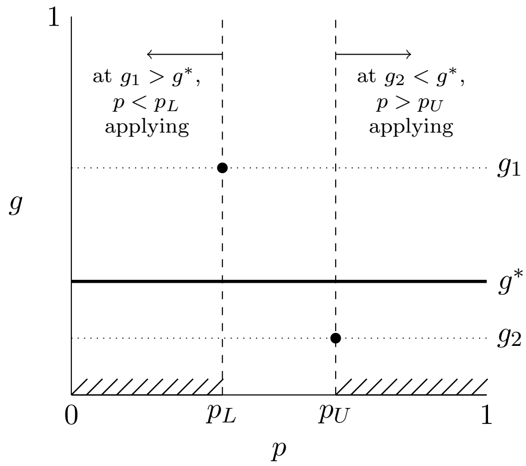

Figure 1: The threshold gearing ratio in a three-period model

Consider gearing ratio $g = g_2 < g^*$, for which the critical level of risk is $p^*(m,g_2) \equiv p_U$. Prospective borrowers with probability of success lower than p_U have reservation rates lower than m and hence postpone their loan applications, whereas lower risk investors have reservation rates higher than m and apply for loans at $t = 0$. This is a conventional effect of uncertainty on investment decision, whereby an increase in uncertainty is associated with greater propensity of risk-neutral investors to postpone capital outlay (McDonald and Siegel, 1986;

Pindyck, 1991; Dixit and Pindyck, 1994). Hence, in the region $g < g^*$, for a given interest rate m, investors apply for loans if their probability of success is above the critical probability p_U. Furthermore, as the interest rate increases, the critical probability level increases and the pool of loan applicants becomes less risky. As a result, interest rate acts as an effective screening device and equilibrium credit rationing does not arise.

5 Conclusion

Positive reservation rate-risk relationship is a necessary condition for equilibrium credit rationing, in the sense of Stiglitz and Weiss (1981), to emerge. If investors can postpone their loan application and hence capital outlay, the sign of the relationship depends on the gearing ratio required to carry out investment. There is a threshold gearing ratio such that, above this threshold, the reservation rate-risk relationship is positive and Stiglitz and Weiss (1981) analysis of equilibrium credit rationing applies. Below the threshold, the relationship between investor's reservation rate and risk is negative and equilibrium credit rationing, in the sense of Stiglitz and Weiss (1981), does not arise.

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